Black holes' quasinormal modes from $\mathcal{N} = 2$ gauge theory and integrability

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Eurostrings 2023, Apr 25th 2023

Based on: arXiv:2208.14031, arXiv:2112.11434, arXiv:1908.08030

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On quantum integrability and $\mathcal{N}=2$ gauge theory

- Broadly speaking, integrability can be considered as the study of **non-linear** phenomena in nature in a quantitative **exact way (non perturbative)**.
- The hallmark of quantum integrability is the presence of **infinite (local) integrals of motion commuting** with each other

$$[\mathbf{I}_{2n-1}, \mathbf{I}_{2m-1}] = 0\,, \tag{1}$$

which are also asymptotic expansion coefficients of the **Baxter's** Q **operator**

$$\ln \mathbf{Q}(\theta) \simeq -C_0 e^{\theta} - \sum_{n=1}^{\infty} e^{\theta(1-2n)} C_n \mathbf{I}_{2n-1} \qquad \theta \to +\infty\,. \tag{2}$$

- Integrable structures appear also in **4D SUSY gauge theories**, typically with $\mathcal{N}=4$ supersymmetry (AdS/CFT correspondence) but also with $\mathcal{N}=2$.
- In $\mathcal{N}=2$ SUSY, the **prepotential** $\mathcal{F}$ is obtained from **gauge periods** a, a_D of Seiberg-Witten differential λ as

$$(a, a_D) = \oint_{A,B} \lambda(x)\, dx\,, \quad a_D = \frac{\partial \mathcal{F}}{\partial a} \quad \Longrightarrow \quad \mathcal{F} \tag{3}$$

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Three different physical theories, same mathematics!

- The **ODE/IM correspondence** allows to derive the integrability structures from some ODE. In particular, the Q **function** (vacuum eigenvalue of Q operator) is defined as

$$Q = W[\psi_+, \psi_-] \qquad \text{with} \qquad \psi_\pm(y) \to 0 \quad y \to \pm\infty\,. \tag{4}$$

$$-\frac{d^2}{dy^2}\psi(y) + \left[2e^{2\theta}\cosh y + P^2\right]\psi(y) = 0 \qquad \text{INTEGRABILITY (sd-Liouville)} \tag{5}$$

$$\frac{\hbar}{\Lambda_0} = \frac{\epsilon_1}{\Lambda_0} = e^{-\theta} \qquad \frac{u}{\Lambda_0^2} = \frac{1}{2}P^2 e^{-2\theta} \qquad \Big\Downarrow \tag{6}$$

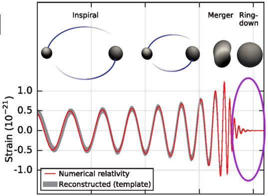
$$-\frac{\hbar^2}{2}\frac{d^2}{dy^2}\psi(y) + [\Lambda^2\cosh y + u]\psi(y) = 0 \qquad \mathcal{N} = 2 \text{ NS GAUGE TH. } (SU(2)\ N_f = 0) \tag{7}$$

$$r = Le^{y/2} \quad \omega L = -2ie^{\theta} \quad P = \frac{1}{2}(l+2) \qquad \Big\Downarrow \qquad \phi(r) = e^{y/2}\psi(y) \tag{8}$$

$$\frac{d^2\phi}{dr^2} + \left[\omega^2\left(1 + \frac{L^4}{r^4}\right) - \frac{(l+2)^2 - \frac{1}{4}}{r^2}\right]\phi(r) = 0 \qquad \text{BLACK HOLES PERT. (D3 brane)} \tag{9}$$

Quasinormal modes in integrability and $\mathcal{N}=2$ gauge theory

- The **quasinormal modes (QNMs)** are the frequencies of the **damped oscillations** in the **ringdown phase** of BHs merging and have a direct connection to **GWs observations**.
- While **computing QNMs** is well understood in General Relativity, in **modified gravity theories** it is still a challenge and it is important to **develop new methods** (analytic and numeric).



- We proved that the **QNMs definition** is a **Bethe root** (zero) condition on the $Q = W[\psi_+, \psi_-]$ function

$$Q(\theta_n) = 0 \qquad \Longleftrightarrow \qquad Q(\theta_n \pm i\pi/2) = \pm i \,. \tag{10}$$

- We proved an **identification of Q function with the gauge period** from which it follows that QNMs are given also by it

$$Q(\theta, P) = \exp \frac{2\pi i}{\hbar} a_D(\hbar, u, \Lambda_0) \implies \frac{1}{\hbar} a_D(i\hbar, -u, \Lambda_0) = \frac{i}{2}\left(n + \frac{1}{2}\right) . \tag{11}$$

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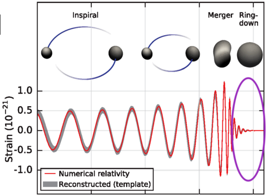

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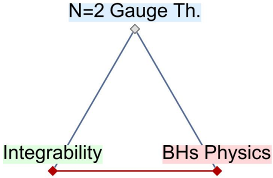

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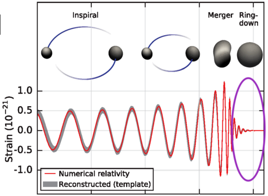

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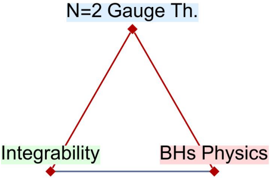

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Computing QNMs from Thermodynamic Bethe Ansatz

- The Q (or $Y = Q^2$) functions satisfy functional equations which can be inverted into the **Thermodynamic Bethe Ansatz (TBA)** for $\varepsilon(\theta) = -2\ln Q(\theta)$ (here for $SU(2)$ $N_f = 0$):

$$\varepsilon(\theta) = \frac{16\sqrt{\pi^3}}{\Gamma(\frac{1}{4})^2} e^{\theta} - 2\int_{-\infty}^{\infty} \frac{\ln\left[1 + \exp\{-\varepsilon(\theta')\}\right]}{\cosh(\theta - \theta')} \frac{d\theta'}{2\pi}\,, \tag{12}$$

with $\varepsilon(\theta, P) \simeq 8P\theta$, $l \sim P > 0$ as $\theta \to -\infty$.

- Through the we have a **new exact method to numerically compute QNMs**, through

$$\varepsilon(\theta_n - i\pi/2) = -i\pi(2n+1)\,. \tag{13}$$

n	l	TBA	Leaver
0	0	$1.36912 - 0.504048i$	$1.36972 - 0.504311i$
0	1	$2.09118 - 0.501788i$	$2.09176 - 0.501811i$
0	2	$2.8057 - 0.501009i$	$2.80629 - 0.501000i$
0	3	$3.51723 - 0.500649i$	$3.51783 - 0.500634i$
0	4	$4.22728 - 0.500453i$	$4.22790 - 0.500438i$

Table: Comparison of QNMs of the D3 brane from TBA (12) (through (13) with $n = 0$), Leaver method (with $L = 1$).

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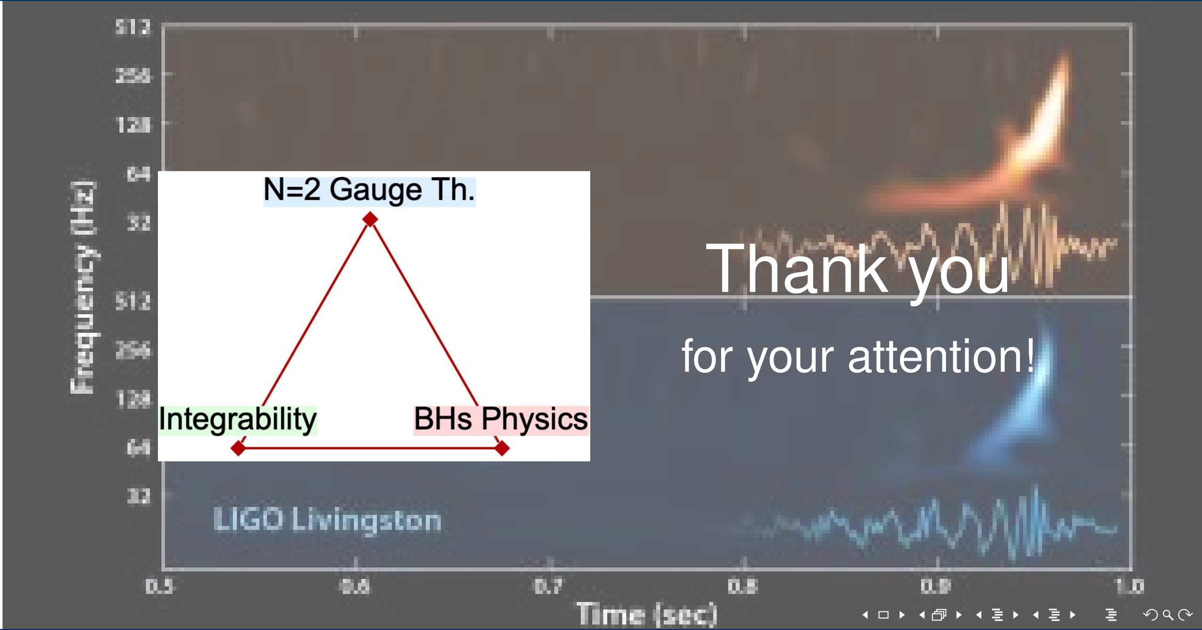

Thank you

for your attention!