Integrability and cycles of deformed $\mathcal{N}=2$ gauge theory

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ODE/IM correspondence

In the **ODE/IM Correspondence**, one can compute and derive all integrability functions (Baxter's Q, Y, T...) and functional relations starting from the solutions of some ODE related to some IM.

Liouville integrable model

The **2D self-dual Liouville IM** is associated to the modified Mathieu equation:

$$\left\{-\frac{d^2}{dy^2}+2e^{2\theta}\cosh y+P^2\right\}\psi(y)=0\,, \qquad (1)$$

$\mathcal{N}=2$ SYM

4D $SU(2)$ $N_f=0$ $\mathcal{N}=2$ SYM, in the Nekrasov Shatasvili (NS) limit of the Ω background ($\epsilon_2\to 0$, $\epsilon_1=\hbar$) is described by the Mathieu equation

$$-\frac{\hbar^2}{2}\frac{d^2}{dz^2}\psi(z)+[\Lambda^2\cos z-u]\psi(z)=0\,. \qquad (2)$$

Gauge-Integrability connection

- The integrability and gauge equations can be related by the **parameters correspondence**

$$\frac{\hbar}{\Lambda}=\frac{\epsilon_1}{\Lambda}=e^{-\theta} \qquad \frac{u}{\Lambda^2}=\frac{1}{2}P^2e^{-2\theta}\,. \qquad (3)$$

- Through ODE/IM, we proved the relations:

$$Q(\theta,P^2)=\exp 2\pi i a_D(\epsilon_1,u,\Lambda)\,, \qquad (4)$$

$$T(\theta,P^2)=2\cos 2\pi a(\epsilon_1,u,\Lambda)\,. \qquad (5)$$

between the two cycle periods a, a_D (leading to the prepotential $\mathcal{F}_{NS}$) of the SYM and the Baxter's Q and T functions of the IM.

Generalizations

- We have evidence that our gauge-integrability connection is of general validity. We proved:
  - $SU(3)$ $N_f=0$ **NS $\mathcal{N}=2$ SYM**, corresponding to **A_2 Toda IM**.
  - $SU(2)$ $N_f=1$ **NS $\mathcal{N}=2$ SYM**, corresponding to **Perturbed Hairpin IM**.

Dictionary of the correspondence

	2D Integrability	4D NS-def. $\mathcal{N}=2$ SYM
1	Q, Y function	(dual) period a_D
2	T function	period a
3	ODE/IM sym.	$\mathbb{Z}_n$ R-symmetry
4	Q, Y system	TBA for gauge period
5	TQ relation	R-symmetry exact rel. for a_D
6	T periodicity	R-symmetry exact rel. for a
7	L.I.M. I_{2n-1}	quantum P.F. eq. for $a^{(n)}, a_D^{(n)}$
8	n-pert. reg. TBA	resum. instanton contr.

From the fundamental identifications 1,2,3 follow 4,5,6,7,8 which are new results for both sides.

References

[1] D. Fioravanti, D. Gregori - arXiv:1908.08030

[2] D. Fioravanti, R. Poghossian, H. Poghosyan - arXiv:1909.11100

[3] D. Fioravanti, D. Gregori, H. Shu - to appear soon